A composite likelihood based approach for max-stable processes using histogram-valued variables

B. Beranger

jointly with T. Whitaker and S. A. Sisson

University of New South Wales, Sydney, Australia

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Motivation (1)

- QUESTION: What is the expected maximum temperature across some region within the next 50 or 100 years?

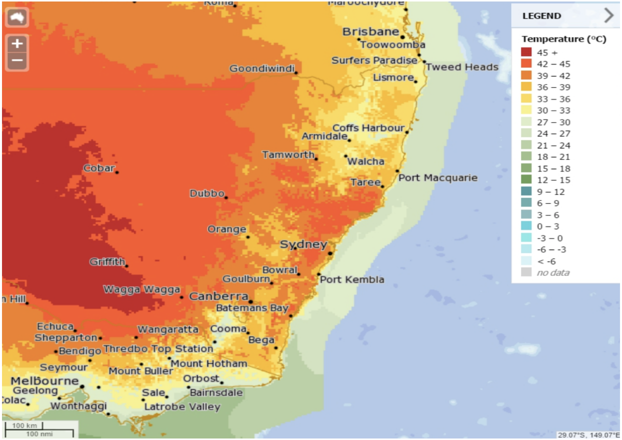

Figure: Heat wave in South East Australia (January 2017)

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  - Unfeasible for a large number of locations and temporal observations

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    $\Rightarrow$ Composite likelihood (Padoan et al. 2010)
  - Unfeasible for a large number of locations and temporal observations
- PROPOSAL: use Symbolic Data Analysis (SDA)

Outline

Max-stable processes (1)

- **Definition:** Let $X_1, X_2, \ldots,$ be i.i.d replicates of $X(s), s \in \mathcal{S} \subset \mathbb{R}^d$.
  If $\exists\, a_n(s) > 0$ and $b_n(s)$, some continuous functions such that
  $$\left\{ \max_{i=1,\ldots,n} \frac{X_i(s) - b_n(s)}{a_n(s)} \right\}_{s\in\mathcal{S}} \xrightarrow{d} \{Y(s)\}_{s\in\mathcal{S}},$$
  then the process $Y(s)$ is a max-stable process with GEV margins.

- Recall that the distribution function of the GEV is given by
  $$G(x;\mu,\sigma,\xi) = \exp\{-v(x;\mu,\sigma,\xi)\},$$
  where $\mu \in \mathbb{R}$, $\sigma > 0$, $\xi \in \mathbb{R}$, $\xi \in \mathbb{R}$, $v(y;\mu,\sigma,\xi) = \left(1 + \xi\frac{y-\mu}{\sigma}\right)_+^{-\frac{1}{\xi}}$ when $\xi \neq 0$ and $e^{-\frac{y-\mu}{\sigma}}$ otherwise, and $a_+ = \min(0, a)$.

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Gaussian extreme value model (Smith, 1990) defined by

$$Y(s) = \max_{1\leq i} \{\zeta_i f_d(s, t_i)\}, s \in \mathbb{R}^d$$

where $(\zeta_i, t_i)_{1\leq i}$ are the points of a point process on $(0, \infty) \times \mathbb{R}^d$, and $f_d = \phi_d(\cdot; \Sigma)$.
For $d = 2$, the bivariate cdf of $(Y(s_1), Y(s_2))$, $s_1, s_2 \in \mathbb{R}^2$ is

$$P(Y(s_1) \leq y_1, Y(s_2) \leq y_2) = \exp\left(-\frac{1}{v_1}\Phi\left(\frac{a}{2} + \frac{1}{a}\log\frac{v_2}{v_1}\right) - \frac{1}{v_2}\Phi\left(\frac{a}{2} + \frac{1}{a}\log\frac{v_1}{v_2}\right)\right),$$

where $v_i = \left(1 - \xi_i \frac{y_i - \mu_i}{\sigma_i}\right)^{-\frac{1}{\xi}}, i = 1, 2$ and $a^2 = (z_1 - z_2)^T \Sigma^{-1} (z_1 - z_2)$

Composite Likelihood (1)

- Let $\mathbf{X} = (X_1, \ldots, X_N)$ denote a vector of N i.i.d. rv's taking values in $\mathbb{R}^K$ with realisation $\mathbf{x} = (x_1, \ldots, x_N) \in \mathbb{R}^{K \times N}$ and density function $g_{\mathbf{X}}(\cdot; \theta)$.

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- Define a subset of $\{1, \ldots, K\}$ by $\mathbf{i} = (i_1, \ldots, i_j)$, where $i_1 < \cdots < i_j$ with $i_j \in \{1, \ldots, K\}$ for $j = 1, \ldots, K-1$.

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- Then for $n = 1, \ldots, N$, $x_n^{\mathbf{i}} \in \mathbb{R}^j$ defines a subset of x_n and $\mathbf{x}^{\mathbf{i}} = (x_1^{\mathbf{i}}, \ldots, x_N^{\mathbf{i}}) \in \mathbb{R}^{j \times N}$, defines a subset of $\mathbf{x}$.

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The **j-wise composite likelihood function, $\mathbf{CL}^{(j)}$** , is given by

$$L_{CL}^{(j)}(\mathbf{x}; \theta) = \prod_{\mathbf{i}} g_{\mathbf{X}^{\mathbf{i}}}(\mathbf{x}^{\mathbf{i}}; \theta),$$

where $g_{\mathbf{X}^{\mathbf{i}}}$ is a $j-$dimensional likelihood function.

Composite Likelihood (2)

- When $j = 2$, the *pairwise* composite log-likelihood function, $l_{CL}^{(2)}$ is given by

$$l_{CL}^{(2)}(\mathbf{x};\theta) = \sum_{i_1=1}^{K-1} \sum_{i_2=i_1+1}^{K} \log g_{\mathbf{X}^{\mathbf{i}}}(\mathbf{x}^{i_1}, \mathbf{x}^{i_2};\theta) \Rightarrow \frac{NK(K-1)}{2} \text{ terms}$$

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- The resulting **maximum j-wise composite likelihood estimator** $\hat{\theta}_{CL}^{(j)}$ is asymptotically consistent and distributed as

$$\sqrt{N}\left(\hat{\theta}_{CL}^{(j)} - \theta\right) \to \mathcal{N}\left(0, G(\theta)^{-1}\right),$$

where $G(\theta) = \mathbf{H}(\theta)\mathbf{J}(\theta)^{-1}\mathbf{H}(\theta)$, $\mathbf{J}(\theta) = \mathbb{V}(\nabla_{CL}^{(j)} l(\theta))$ is a variability matrix and $\mathbf{H}(\theta) = -\mathbb{E}(\nabla_{CL}^{2(j)} l(\theta))$ is a sensitivity matrix.

Symbolic likelihood function

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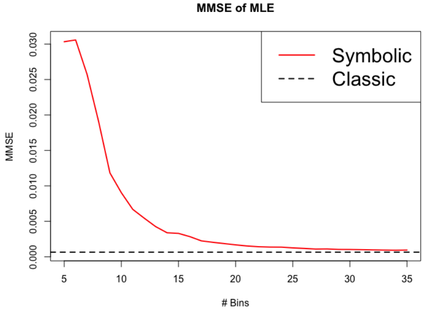
- From now on S is assumed to take a **histogram** as value.



Histogram-valued symbols (1)

- Let $D_X = \mathbb{R}^K$, the classical data $\mathbf{x} \in \mathbb{R}^{N \times K}$ can be aggregated into a K-dimensional histogram with $B^1 \times \cdots \times B^K$ bins

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- Denote the **bin index** by $\mathrm{b} = (b_1, \ldots, b_K)$, $b_k = 1, \ldots, B^k$, $k = 1, \ldots, K$. A **bin** b is given by $\Upsilon_{\mathrm{b}} = \Upsilon^1_{b_1} \times \cdots \times \Upsilon^K_{b_K}$, where $\Upsilon^k_{b_k} = (y^k_{b_k - 1}, y^k_{b_k}]$, $y^k_{b_k} \in \mathbb{R}$ are fixed

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- $\mathbf{s} = (s_{\mathbf{1}}, \ldots, s_{\mathbf{B}})$ gives the observed numbers of counts in the bins $\mathbf{1} = (1, \ldots, 1)$ up to $\mathbf{B} = (B^1, \ldots, B^K)$. It is a vector of size $B^1 \times \cdots \times B^K$, verifying $\sum_{\mathrm{b}} s_{\mathrm{b}} = N$.

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The histogram symbolic likelihood function is then written as

$$L(\mathbf{s}; \theta) = \frac{N!}{s_{\mathbf{1}}! \ldots s_{\mathbf{B}}!} \prod_{\mathrm{b}=\mathbf{1}}^{\mathbf{B}} P_{\mathrm{b}}(\theta)^{s_{\mathrm{b}}}$$

where $P_{\mathbf{b}}(\theta) = \int_{\Upsilon_{\mathbf{b}}} g_X(x; \theta) dx$. Note that g_X is a K-dim density.

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The symbolic likelihood function associated with the vector of counts $\mathbf{s}_j^{\mathbf{i}} = (s_{\mathbf{1^i}}^{\mathbf{i}}, \ldots, s_{\mathbf{B^i}}^{\mathbf{i}})$ of length $B^{i_1} \times \cdots \times B^{i_j}$ is

$$L(\mathbf{s}_j^{\mathbf{i}}; \theta) = \frac{N!}{s_{\mathbf{1^i}}^{\mathbf{i}}! \cdots s_{\mathbf{B^i}}^{\mathbf{i}}!} \prod_{\mathrm{b}^{\mathbf{i}}=\mathbf{1^i}}^{\mathbf{B^i}} P_{\mathrm{b}^{\mathbf{i}}}(\theta)^{s_{\mathrm{b}^{\mathbf{i}}}^{\mathbf{i}}},$$

where $P_{\mathrm{b}^{\mathbf{i}}}(\theta) = \int_{\Upsilon_{b_{i_1}}^{i_1}} \ldots \int_{\Upsilon_{b_{i_j}}^{i_j}} g_X(x;\theta)dx$ and g_X is a $j-$dim density.

Histogram-valued symbols (3)

- $\mathbf{s}_j = \{\mathbf{s}_{jt}^{\mathbf{i}}; t = 1, \ldots, T, \mathbf{i} = (i_1, \ldots, i_j), i_1 < \ldots < i_j\}$ represents the set of $j-$dimensional observed histograms for the symbolic-valued random variable $\mathbf{S}_j$

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- The **symbolic $j-$wise composite likelihood function ($\mathbf{SCL}^{(j)}$)** is given by

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$$L_{SCL}^{(j)}(\mathbf{s}_j; \theta) = \prod_{t=1}^{T} \prod_{\mathbf{i}} L(\mathbf{s}_{jt}^{\mathbf{i}}; \theta)$$

- Components of the Godambe matrix are given by

$$\hat{H}(\hat{\theta}_{SCL}^{(j)}) = -\frac{1}{N} \sum_{t=1}^{T} \sum_{\mathbf{i}} \nabla^2 l(\mathbf{s}_{jt}^{\mathbf{i}}; \hat{\theta}_{SCL}^{(j)})$$

$$\hat{J}(\hat{\theta}_{SCL}^{(j)}) = \frac{1}{N} \sum_{t=1}^{T} \left(\sum_{\mathbf{i}} \nabla l(\mathbf{s}_{jt}^{\mathbf{i}}; \hat{\theta}_{SCL}^{(j)}) \right) \left(\sum_{\mathbf{i}} \nabla l(\mathbf{s}_{jt}^{\mathbf{i}}; \hat{\theta}_{SCL}^{(j)}) \right)^{\top}$$

The simulation set up

- K locations are generated uniformly on a $(0, 40) \times (0, 40)$ grid
- N realisations of the Smith model are generated for each location
- MLE's are obtained using $\mathrm{CL}^{(2)}$ and $\mathrm{SCL}^{(2)}$

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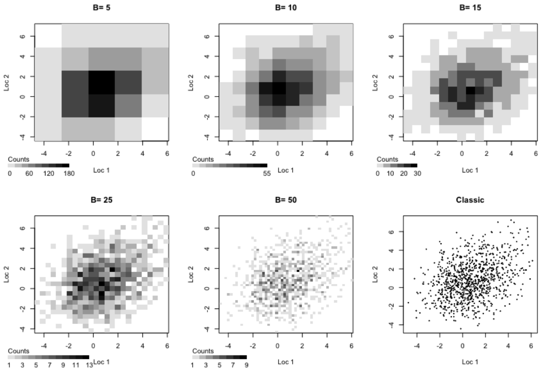

Experiement 1 - Increasing the number of bins

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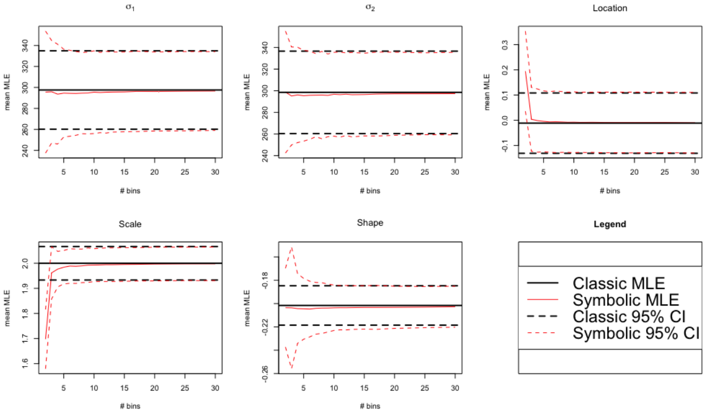

Figure: Mean of MLEs for $\theta = (\sigma_{11}, \sigma_{12}, \sigma_{22}, \mu, \sigma, \xi)$ using $\mathrm{CL}^{(2)}$ and $\mathrm{SCL}^{(2)}$, for increasing number of bins in bivariate histograms.

Experiement 2 - Computation time

- $B = 25,\ K = 10, 100,\ T = 1,$ Repetitions $= 10$

	$K = 10$			$K = 100$		
N	t_c	t_s	t_{hist}	t_c	t_s	t_{hist}
100	9.8	18.6	0.7	9758.6	1594.5	72.3
500	27.6	26.2	0.8	45040.1	2218.8	74.2
1000	71.9	22.5	0.8	-	2238.0	78.8
5000	291.8	19.0	0.8	-	2650.2	81.7
10000	591.7	23.8	0.9	-	2356.6	85.8
50000	2626.8	24.2	1.7	-	2300.6	131.6
100000	5610.7	25.4	2.4	-	2766.9	188.2
500000	31083.1	23.2	7.5	-	3111.5	627.1

Table: Mean computation times (sec) to optimise the regular and symbolic composite likelihood (t_c and t_s), and to aggregate the data into bivariate histograms (t_{hist})

Experiement 3 - Convergence of variances (1)

- $B = 25$, $N = 1000$, $K = 10$, Number of repetitions $= 1000$

T	σ_{11}	σ_{12}	σ_{22}	μ	σ	ξ
4	226.93	97.63	167.27	0.105	0.051	0.030
5	203.04	87.36	149.66	0.095	0.047	0.028
10	143.92	61.95	106.04	0.071	0.036	0.021
20	102.23	44.04	75.27	0.054	0.029	0.016
40	72.93	31.48	53.64	0.043	0.024	0.013
50	65.52	28.31	48.16	0.040	0.023	0.012
100	47.38	20.55	34.71	0.034	0.020	0.011
200	34.87	15.23	25.42	0.030	0.018	0.010
1000	21.12	13.08	13.11	0.025	0.016	0.010
Classic	16.65	10.53	10.69	0.020	0.014	0.009

Table: Mean variances calculated from $CL^{(2)}$ and $SCL^{(2)}$ for $\theta = (\sigma_{11}, \sigma_{12}, \sigma_{22}, \mu, \sigma, \xi)$ for increasing T.

Experiement 3 - Convergence of variances (2)

- $\hat{J}(\hat{\theta}^{(j)}_{SCL})$ requires $T \to N$ and $\mathbf{B} \to \infty$ for the convergence towards the classical Godambe matrices to occur.

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- $\hat{J}(\hat{\theta}^{(j)}_{SCL})$ requires $T \to N$ and $\mathbf{B} \to \infty$ for the convergence towards the classical Godambe matrices to occur.
- For T fixed, convergence still occurs as $\mathbf{B} \to \infty$ towards a different expression

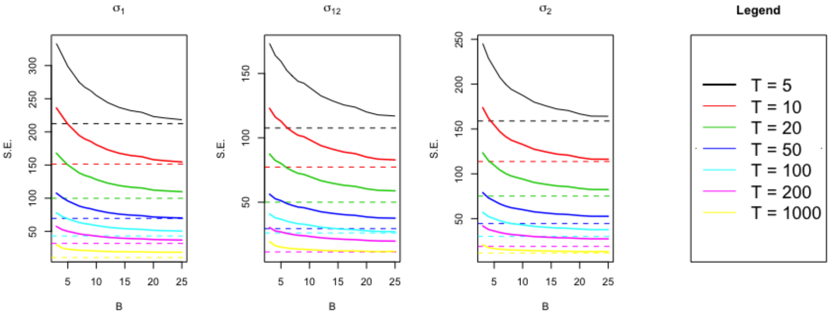

Figure: Mean variances calculated from SCL$^{(2)}$ for fixed T and increasing $\mathbf{B}$.

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- **Data:**
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- **Data:**
  - Focus on fortnighly maxima at $K = 105$ locations over summer months
  - 3 sets: historical ($N = 970$), RCP4.5 and RCP8.5 (both $N = 540$)
- Bivariate histograms are constructed for all pairs of locations for $B = 15, 20, 25, 30$.

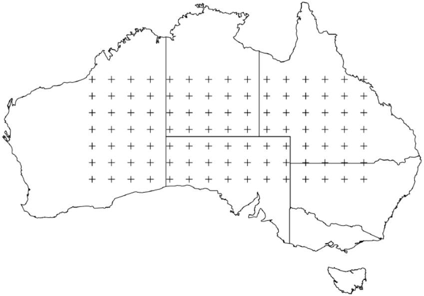

Figure: Study region

Model fitting

- Fit the Smith model with mean and variance parameters as linear functions of space

	σ_{11}	σ_{12}	σ_{22}	ξ
B	Historical Data			
15	176.4 (0.285)	-28.7 (0.032)	76.8 (0.329)	-0.266 (0.053)
20	164.2 (0.289)	-29.3 (0.030)	74.3 (0.469)	-0.264 (0.049)
25	162.4 (0.217)	-29.9 (0.033)	75.3 (0.284)	-0.264 (0.049)
30	161.6 (0.201)	-32.3 (0.029)	74.4 (0.234)	-0.264 (0.050)
B	RCP4.5 Data			
15	160.9 (0.942)	-34.1 (0.083)	79.0 (0.222)	-0.249 (0.074)
20	163.5 (0.595)	-41.1 (0.073)	77.6 (0.245)	-0.249 (0.076)
25	150.3 (0.349)	-33.1 (0.065)	70.7 (0.170)	-0.250 (0.073)
30	150.2 (0.150)	-31.6 (0.024)	70.7 (0.154)	-0.250 (0.069)
B	RCP8.5 Data			
15	128.7 (0.860)	-19.6 (0.092)	67.7 (0.392)	-0.232 (0.061)
20	128.0 (0.630)	-19.6 (0.129)	66.6 (0.332)	-0.231 (0.059)
25	136.0 (0.395)	-15.1 (0.093	59.4 (0.317)	-0.234 (0.060)
30	129.9 (0.401)	-13.6 (0.083)	56.4 (0.294)	-0.233 (0.055)

Figure: MLEs using the $SCL^{(2)}$ for various values of B.

Estimated location parameter

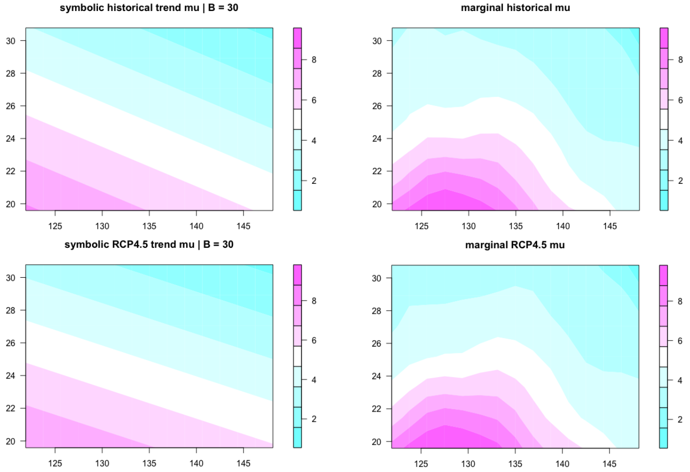

Figure: Estimated surfaces for the location parameter using the $l_{SCL}^{(2)}$ function (left) and marginal GEV estimations (right)

Examples of return level plots

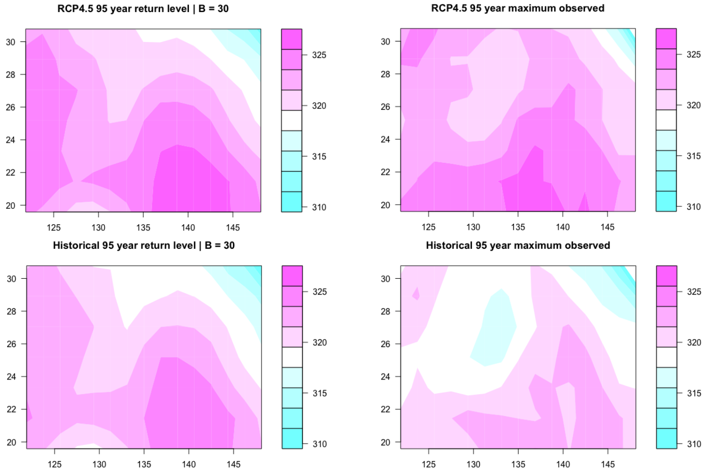

Figure: Estimated 95 year return levels using the $l_{SCL}^{(2)}$ function (left) and observed 95 year return levels (right)

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- **Open questions:**
  - How do we choose our symbols?
  - What is a sufficient B such that the symbolic results have converged to the classical results?

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- The symbolic composite likelihood function presents similar asymptotic properties to that of the classical composite likelihood function
- **Open questions:**
  - How do we choose our symbols?
  - What is a sufficient B such that the symbolic results have converged to the classical results?

THANK YOU!

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