Extremes Without Borders:

Multivariate and Spatial Perspectives

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SSA NSW Branch, Lancaster lecture
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Henry O. Lancaster

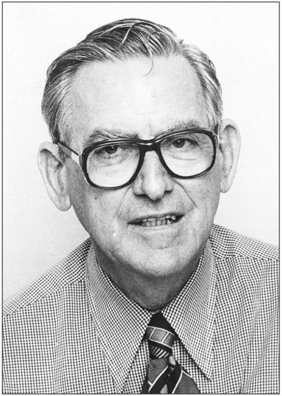
Extremes are rare...but everywhere!

Extremes Value Theory: a review of concepts

Contribution(s) to the field

Henry Oliver Lancaster, AO FAA (1 Feb 1913 – 2 Dec 2001)

Biography



- Tertiary education: University of Sydney
  1930: Economics (4 weeks) and then Arts
  1931–37: Medicine (MB BS)
- 1938: Pathologist and Senior Medical Officer at Sydney Hospital
- 1940–46: Medical Officer in the Australian Imperial Force (Middle East and New Guinea)
- 1945: Honours in Pure Maths (USyd)
- 1946: Lecturer in Medical Statistics

Henry Oliver Lancaster, AO FAA (1 Feb 1913 – 2 Dec 2001)

- 1948: Rockefeller Fellow in Medicine at the London School of Hygiene.
- 1953: PhD "*The Application of Chi-Squared to Discrete Distributions*".
- 1959: Associate Professor in the School of Public Health and Tropical Medicine.
- 1959–78: Foundation Chair in Mathematical Statistics. Established the new Department of Mathematical Statistics (with Harry Mulhall).
- 1961: Elected fellow of the Australian Academy of Science.
- 1978: Emeritus Professor at USyd.
- 1980: Pitman Medal of the SSA.
- Produced over 130 research papers and six books covering both his medical and mathematical work

Lancaster & the SSA

- 1947: Lancaster, Rutherford and Turner founded the **Statistical Society of New South Wales** which later became the **Statistical Society of Australia** (1962).
- 1949–58: Editor of *The Bulletin of the Statistical Society of New South Wales*
- 1959–71: Appointed as the founding editor of *The Australian Journal of Statistics*

Lancaster & the Australian Mathematical Society

- 1956: Elected to the first council
- 1959–63: Secretary
- 1966–67: President

Henry O. Lancaster

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1953 North Sea floods

On night of 31 Jan–1st Feb 1953 **an extreme storm tide** in the North Sea was caused by a **combination** of:

- high spring tide
- severe European windstorm.

Water levels **locally exceeded 5.6m above (mean) sea level**:

- overwhelmed sea defences
- flooded areas of the Netherlands, England, Belgium, Denmark and France
- $\sim$2,500 people died.

Watersnoodramp, 1953

1953 North Sea floods

Dutch government set up **Delta Commission**.

Set up flood defences such that the **acceptable period over which complete failure would occur** is:

- North and South Holland:
  1 per 10,000 years
- Other areas at risk from sea flooding:
  1 per 4,000 years
- Transition between high and low land:
  1 per 2,000 years

Erith, London, 1953

Based on data observed over a much shorter period!!

1953 North Sea floods

Maeslantkering, South Holland

Thames Barrier, London

Defences completed in 1997.

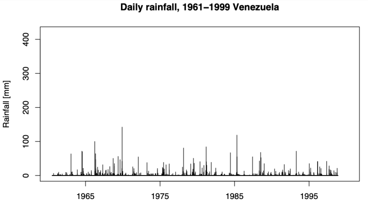
1999 The Vargas tragedy

On 14–16 December 1999 **torrential rains caused landslides** in Vargas State, on the northern coast of Venezuela.

Major natural catastrophe:

- Caused flash floods and debris flows, mud flows
- Killed tens of thousands of people (10% of Vargas State population)
- Completely destroyed (gone) many coastal tourist towns

https://en.wikipedia.org/wiki/Vargas_tragedy

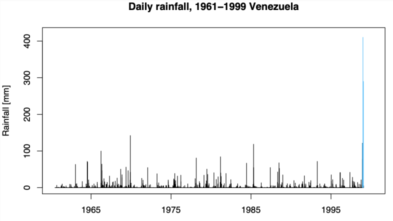


Daily rainfall at Maiquetia International Airport

1999 The Vargas tragedy

The 3-day storm:

- Day 1 = any previously seen max
- Day 2 >3 times previous max
- Day 3 >2 times previous max



Estimated recurrence time of the 3-day storm range from **250–6 million years!**

How to estimate this? How can we be so uncertain?!

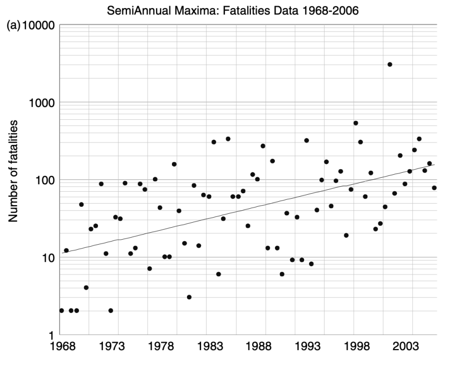
Again: Need to extrapolate events beyond the observable data.

2009 The risk of terrorism

Unlike natural disasters, “terrorist threat” is less well understood.

Want to quantify for risk assessment, event prevention, insurance and financial markets.

“Terrorist act” measured by number of fatalities (or casualties).
$\rightarrow$ Maximum daily fatalities over 6 months.



2009 The risk of terrorism

Several challenges:

- Clear non-stationarity in the data
  E.g. 9/11 was a clear extreme in 2001, but it is less unlikely now.
- Event generation mechanism is non-stationary
  E.g. as a response to other events that may (not) be occurring.
- Data are discrete
- Influenced by many factors
  E.g. country of event, nationality of perpetrators etc.

If (X_t, Y_t) are (max fatalities, max casualties) for 6-month period t, and want to model jointly, note that X_t and Y_t may correspond to different actual events.


Is this a problem?

https://www.start.umd.edu/gtd/

2009 Black Saturday bushfires

Facts:

- On 7 February 2009, in Victoria.
- 173 fatalities, 400 injured.
- Destroyed more than 2,000 homes
- Driven by extreme heat, drought, and wind shifts.
- Creates firestorms and even pyrocumulonimbus clouds.

2021 Bitcoin prices

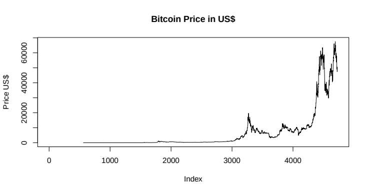
Let X_t be the closing price of a quantity on day t, it is common to model the log daily returns:

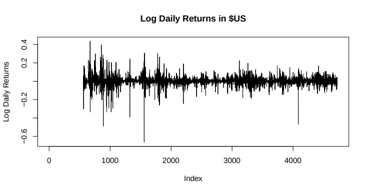
$$Y_t = \log(X_t/X_{t-1}).$$

To quantify risk, interest is in extremely large/small log daily returns.

Plots show 4,727 observations of

- Closing price (X_t) in US$
- Log daily returns (Y_t)
- Covering 3rd Jan 2009 – 12th Dec 2021.





Extreme high/low changes in Y_t don't correspond to obvious big changes in X_t.

How to model extreme high/low changes in Y_t?

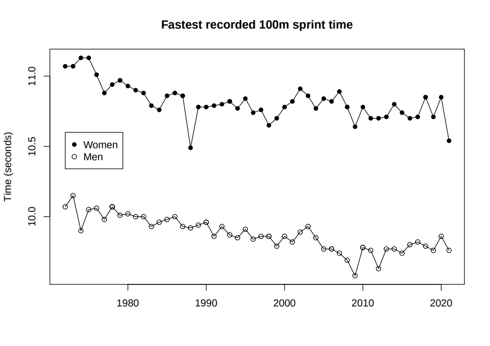
2021 Fastest 100m sprint times

Fastest 100m times show clear improvement since the 1970s.

Possibly constant performance since ∼1990 (women) and ∼2005 (men)?

Various factors affect these times (wind speed & direction, hand timing etc).

Have we reached the limit of performance?



What is the probability that:

- The world record will be broken next year?
- The world record will hold for the next 50 years?

https://en.wikipedia.org/wiki/100_metres

2022 Northern NSW Floods (Lismore)

Facts:

- Late February – early March 2022
- Wilsons River floodplain
- Record river height: ~ 14.4m
- Rainfall: > 700 mm in ~ 3 days in parts of the catchment
- Thousands of homes inundated
- insured losses AUD $6 - 7+$ billions

This is a textbook compound extreme:

1. Large-scale climate conditions: La Niña phase (wetter eastern Australia) and warm ocean temperatures (enhanced moisture)
2. East Coast Low: persistent rainfall over same region
3. Orographic effects: low elevation, broad catchment, Saturated soils

Other major events

- 1983 **Ash Wednesday fires**: 75 deaths.
- 1983 **Melbourne dust storm**: $\sim 50,000$ tonnes of topsoil displaced, linked to severe drought + strong winds.
- 1990-91 **Cyclone Joy**: wind + rainfall + storm surge. Also **Cyclone Yasi** (2011) and **Narelle** (2026).
- 2005 **Hurricane Katrina**: biggest insured-loss extreme event, USD 65-90 billions.
- 2007-09 **Global Financial Crisis**: Massive negative returns across global markets.
- 2019–20 **Black Summer fires**: massive spatial extent.

Take away message

- Extremes are impactful in many fields
- Need to extrapolate beyond observed data
- Marginal extremes still matter – but are not enough (Rainfall totals in Lismore, Peak river height, Market returns in GFC)
- Extremes rarely occur in isolation, dependence is central
  - Cyclones $\rightarrow$ wind + rain + surge
  - GFC $\rightarrow$ joint crashes across assets
- Spatial extent drives impact
  - Bushfires and heatwaves cover large regions
  - Floods affect entire catchments
- Non-stationarity is unavoidable (changes in time and space)

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Contribution(s) to the field

Univariate extremes

Assume $X_1, \ldots$ is a sequence of i.i.d. random variables with **common distribution function F**.

How to define and model extremes?

Approach 1: Block Maxima

Consider the **maximum of a "block"** of size n:

$$M_n = \max_{i=1,\ldots,n} X_i$$

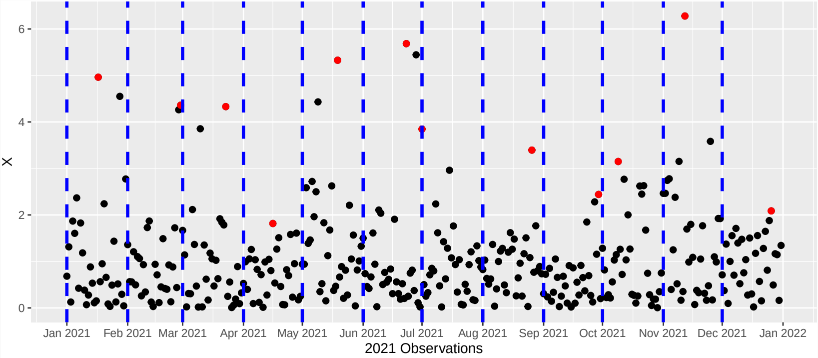
Assuming a dataset of size $n \times m$, it is divided into **m blocks** of **size n**. We obtain a sequence of (only) m maxima that we can define as

$$\left(M_n^1, \ldots, M_n^m\right), \quad M_n^j = \max_{i \in \text{ bloc } j} X_i$$

Univariate extremes

Example

Consider the daily concentration level of an air pollutant in 2021 ($n \times m = 365$). Focus on the monthly max concentration ($m = 12$).



Simulated daily levels of an air pollutant in 2021. Red dots show monthly maxima.

Univariate extremes

Assume that there exists sequences of constants $(a_n) > 0$, $(b_n) \in \mathbb{R}$, and a non-degenerate random variable Y with d.f. G, such that

$$\frac{M_n - b_n}{a_n} \xrightarrow{d} Y, \quad \text{as } n \to \infty.$$

Theorem[**Fisher & Tipett, 1928; Gnedenko 1943**]
The only possible limiting distribution function G, up to location-scale transformation, is the **Generalized Extreme Value (GEV)** distribution. Write $G_\xi(y; \mu, \sigma)$.

Approach 2: Peaks over Threshold

Consider the events that **exceed some high threshold** $u < x^*$, where x^* is the right end point of X.

Theorem [Pickands, 1975; de Haan & Balkema, 1974]

Let $X_i, i = 1, \ldots, n$ with common df F such that

$$\mathbb{P}(M_n \leq y) \approx G_\xi(y; \mu, \sigma).$$

Then, **for large enough** u,

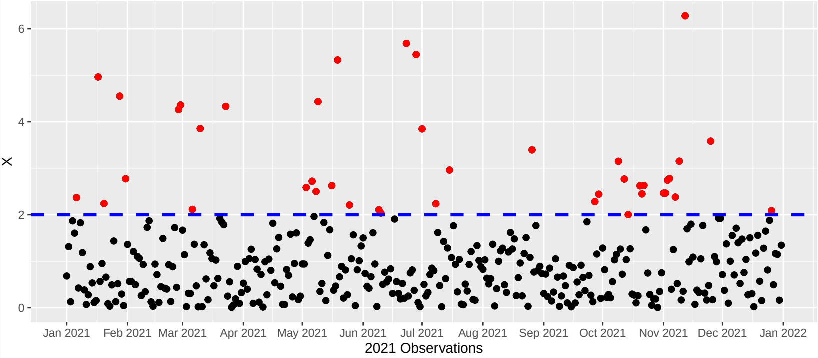
$$\mathbb{P}\left(X - u \leq y | X > u\right) \approx H_\xi(y; \tilde{\sigma}),$$

where H represents the Generalized Pareto distribution and $\tilde{\sigma} = \sigma + \xi(u - \mu)$.

Univariate extremes

Example

Consider the threshold $u = 2$: there are 42 observations above this level against 12 monthly maxima.



Simulated daily levels of an air pollutant in 2021. Red dots represent the observations above the threshold $u = 2$.

Multivariate extremes

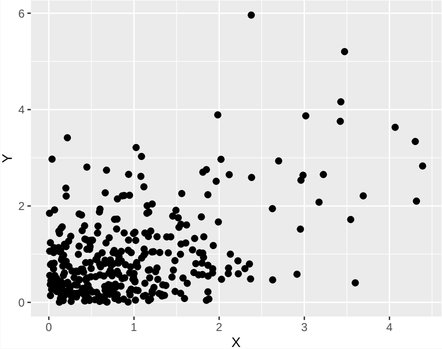
Example $d = 2$

Consider the daily concentration level of **two** air pollutants X and Y in 2021 ($n \times m = 365$).

We lose the temporal element in the scatterplot!

How to define bivariate blocks?

Short answer: We can't!



Multivariate extremes

Approach 1: Componentwise maxima

Define **marginal blocks** $M_{n,j} = \max_{i=1,\dots,n} X_{i,j}$ and the associated **vector of componentwise maxima**

$$\boldsymbol{M_n} = (M_{n,1}, \dots, M_{n,d}).$$

This may not correspond to an actual observation!

Example $d = 2$

We have $\boldsymbol{M_n} = (M_{n,x}, M_{n,y})$ and the observed data matrix is

$$\begin{pmatrix} \max_{i \in \text{Jan 2021}} X_i & \max_{i \in \text{Jan 2021}} Y_i \\ \vdots & \vdots \\ \max_{i \in \text{Dec 2021}} X_i & \max_{i \in \text{Dec 2021}} Y_i \end{pmatrix}.$$

Multivariate extremes

Asymptotic distribution of M_n

Assume there exists sequences of normalising constants $\boldsymbol{a_n} = (a_{n,1}, \ldots, a_{n,d}) > \mathbf{0} = (0, \ldots, 0)$ and $\boldsymbol{b_n} = (b_{n,1}, \ldots, b_{n,d}) \in \mathbb{R}^d$ such that

$$\mathbb{P}\left(\frac{\boldsymbol{M_n} - \boldsymbol{b_n}}{\boldsymbol{a_n}} \leq \mathbf{y}\right) = F^n(\boldsymbol{a_n}\mathbf{y} + \boldsymbol{b_n}) \xrightarrow{n\to\infty} G(\mathbf{x}),$$

for all continuity points $\mathbf{x}$ of a non-degenerate distribution G called **multivariate extreme value distributions (MEVDs)**.

The marginal distributions are GEV and assuming standardisation to unit Fréchet margins, we have

$$G(z_1, \ldots, z_d) = \exp\left\{-\mathbf{V}(\mathbf{z_1}, \ldots, \mathbf{z_d})\right\},$$

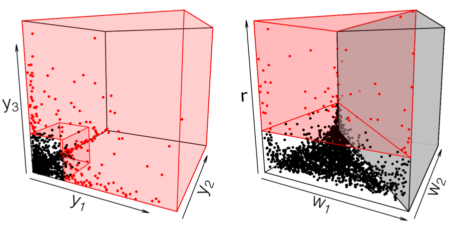
where the V is called the exponent function which can be characterised by the spectral measure H (details omitted).

Example - Independence

If H is not differentiable but places mass $1/d$ on the d-dimensional vectors $\boldsymbol{e}_j = (0,\ldots,0,1,0,\ldots,0)$, then

$$V(z_1,\ldots,z_d) = \frac{1}{z_1} + \ldots + \frac{1}{z_d} = \sum_{j=1}^{d} \frac{1}{z_j},$$

and thus
$$G(z_1,\ldots,z_d) = \prod_{j=1}^{d} \exp\left\{-\frac{1}{z_j}\right\}$$



Example - Perfect dependence

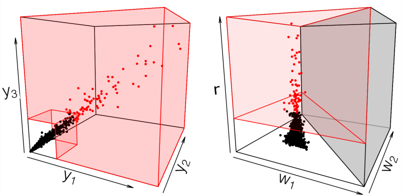
If H is not differentiable but places mass 1 on $\boldsymbol{\omega} = \left(\frac{1}{d}, \ldots, \frac{1}{d}\right)$, then

$$V(z_1, \ldots, z_d) = d \times \max\left(\frac{1}{dz_1}, \ldots, \frac{1}{dz_d}\right),$$

which implies that

$$G(z_1, \ldots, z_d) = \exp\left\{-\max\left(\frac{1}{z_1}, \ldots, \frac{1}{z_d}\right)\right\}$$

joint distribution of variables that are marginally standard Fréchet, but which are perfectly dependent: $X_1 = \ldots = X_d$ with probability 1.



Multivariate extremes

Approach 2: Multivariate Peaks over Thresholds
Consider the events for which at least one component j exceeds a high threshold u_j.

Theorem **[Rootzen et al., 2018]**
Assume $\mathbf{X} \nleq \boldsymbol{u}$ indicates $\mathbf{X} \leq \boldsymbol{u}$ does not hold, and assume $\mathbf{X}$ has unit Pareto margins, then

$$\mathbf{X} - \boldsymbol{u} \Big| \mathbf{X} \nleq \boldsymbol{u} \xrightarrow{d} \tilde{\mathrm{Y}}, \quad \text{as } \boldsymbol{u} \to \infty,$$

where $\tilde{\mathrm{Y}}$ follows a multivariate Generalized Pareto distribution that is fully characterized by the V function.

Spatial extremes

We now study the extremes of a stochastic process $X(s), s \in \mathcal{S}$

Approach 1: Max-stable processes

Assume there exists some continuous functions $a_n(s) > 0$ and $b_n(s)$ such that

$$\left\{ \max_{i=1,\ldots,n} \frac{X_i(s) - b_n(s)}{a_n(s)} \right\}_{s \in \mathcal{S}} \xrightarrow{d} \{Z(s)\}_{s \in \mathcal{S}}$$

Definition [Schlather, 2002] A max-stable process with unit Fréchet margins can be characterized as

$$Z(\mathbf{s}) = \sup_{i=1}^{\infty} R_i W_i(\mathbf{s}), \ \mathbf{s} \in \mathcal{S},$$

where $R_1, R_2, \ldots,$ are the points of a PPP on $(0, \infty)$ and $W_1(\mathbf{s}), W_2(\mathbf{s}), \ldots,$ are independent copies of $W(\mathbf{s})$ with unit mean.

Spatial extremes

The exponent measure restricted onto $\mathbb{R}_+^D$ is given by

$$\kappa\left([\mathbf{0}, \boldsymbol{x}]^c\right) = \int_0^\infty 1 - \Pr(\boldsymbol{W} \in [\mathbf{0}, \boldsymbol{x}r])\mathrm{d}r, \quad \mathbf{x} \in \Omega,$$

where $\boldsymbol{W} = (W(\mathbf{s}_1), \ldots, W(\mathbf{s}_D))^\top$ and $\Omega = \mathbb{R}_+^D \setminus \{\mathbf{0}\}$.

The distribution function can be expressed as

$$G(\boldsymbol{x}) = \exp\left\{-\kappa([0, \boldsymbol{x}]^c)\right\} = \exp\left\{-V(\boldsymbol{x})\right\}.$$

Approach 2: r-Pareto processes

The exceedances are now defined through a risk function $r(\cdot)$, which allows to select different types of events.

Definition **[Dombry & Ribatet, 2015]** Assuming the process X with unit Pareto margins satisfying $\lim_{u\to\infty} u \Pr(X/u \in B) = \kappa(B), \forall B \subset C^+(\mathcal{S})$, then the limiting process

$$\tilde{Z}(\mathbf{s}) = \lim_{u\to\infty} \frac{X(\mathbf{s})}{u} | r\left(\{X(\mathbf{s}), \mathbf{s} \in \mathcal{S}\}\right) > u,$$

defines a simple r-Pareto process on $\mathcal{A}_r = \{f \in C^+(\mathcal{S}) : r(f) > 1\}$.

Spatial extremes

The probability measure of the r-Pareto process is given by $\kappa(\cdot \cap \mathcal{A}_r)/\kappa(\mathcal{A}_r)$.

The finite dimensional density is therefore

$$\frac{\kappa(\mathbf{x})}{\kappa(\mathcal{A}_r^D)}, \quad \mathbf{x} \in \mathcal{A}_r^D,$$

where κ is the intensity function and $\mathcal{A}_r^D$ is the set $\mathcal{A}_r$ restricted to D dimensions.

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Contribution(s) to the field

(Some) Collaborators

Pavel Krupskii
Melbourne

Simone Padoan
Bocconi

Scott Sisson
UNSW

Peng Zhong
Wollongong

Back to Max-stable processes

Recall $G(\boldsymbol{x}) = \exp\{-V(\boldsymbol{x})\}$, $\boldsymbol{x} \in \Omega = \mathbb{R}_+^D \setminus \{\mathbf{0}\}$

Let $B_D = \{1, \dots, D\}$ and $B_k = \{b_1, \dots, b_k\} \subset B_D$, where $b_1 < \cdots < b_k$.

Let $\Omega_{B_k} = \{\boldsymbol{x} \in \Omega : x_j = 0 \text{ if } j \notin B_k\}$ such that:

- $\partial\Omega = \{\Omega_{B_k}, \forall B_k \text{ and } k = 1, \dots, D-1\}$ represents the boundaries of Ω,
- $\Omega^\circ = \Omega \setminus \partial\Omega$ denotes the Interior of Ω.

Important. Depending on the choice of W, the exponent measure κ **can put mass on both $\partial\Omega$ and Ω°** with the intensity function on each subspace Ω_{B_k}

$$\lim_{x_i \to 0, i \notin B_k} -V_{B_k}(\boldsymbol{x}), \quad V_{B_k} = \frac{\partial^k V}{\partial x_{b_1} \dots \partial x_{b_k}}.$$

On Ω°, it can be expressed as $\kappa(\boldsymbol{x}) = -V_{B_D}(\boldsymbol{x})$, where the function κ is referred to as the intensity function of the max-stable process.

Max-stable processes - Inference

Full likelihood: intractable!

Composite likelihood: Popular but still limited.

Stephenson-Tawn likelihood: Can be biased, moderate dimensions.

Spectral likelihood [Coles & Tawn (1991)] If data $\in$ MDA($\boldsymbol{Z}$) then can be approximately treated as points of a PPP with measure $\kappa(\cdot)$. For a model with parameter $\boldsymbol{\theta}$ and high u, the log-likelihood is

$$\ell_A(\boldsymbol{\theta}; \mathbf{x}_1, \ldots, \mathbf{x}_n) \propto \sum_{i \in \{m : \|\mathbf{x}_m\|_1 > u\}} \log \kappa\left(\boldsymbol{x}_i; \boldsymbol{\theta}\right).$$

This requires convergence of:

a) X to the max-stable process Z by taking pointwise maxima.

b) X to the Poisson point process.

The fact that κ can put mass on $\partial\Omega$ hinders the convergence of $X \Longrightarrow$ bias.

r-Pareto processes - Inference

The log-likelihood is thus

$$\ell_{rP}(\boldsymbol{\theta}; \mathbf{x}_1, \ldots, \mathbf{x}_n) = \sum_{i \in \{m : r(\mathbf{x}_m) > u\}} \log\left(\frac{\kappa(\mathbf{z}_i; \boldsymbol{\theta})}{\kappa(\mathcal{A}_r; \boldsymbol{\theta})}\right),$$

where $\mathbf{z}_i = \mathbf{x}_i / u$ represent the realizations of the r-Pareto process.

Important.

- $\kappa(\mathcal{A}_r; \boldsymbol{\theta})$ involves integration over $\mathbb{R}_+^D$, $\Longrightarrow$ intractability
- Score matching approach by de Fondeville & Davison (2018):
- $r(\mathbf{x}) = \|\mathbf{x}\|_1 \Longrightarrow$ spectral likelihood.
- If the exponent measure κ has discontinuities (presence of mass on $\partial\mathcal{A}_r^D$), $\Longrightarrow$ Inference requires evaluation of $-V_{B_k}(\boldsymbol{x})$.
  - ⋆ Restriction to the Brown-Resnick models

Ensuring continuous exponent measures

Theorem 1. [Zhong, Sisson & Béranger (2025)]
Consider the max-stable process $\{Z(\mathbf{s}), \mathbf{s} \in \mathcal{S}\}$ defined at D locations and assume the partial derivatives of the function V exist.

The intensity function on $\partial\Omega$ is zero almost everywhere if and only if the conditional probability of $\boldsymbol{W}$ satisfies

$$\Pr(\boldsymbol{W}_{\bar{B}_k} = \mathbf{0}_{D-k} \mid \boldsymbol{W}_{B_k} = \boldsymbol{x}_{B_k}) = 0, \ \forall\, k \in \{1, \ldots, D-1\},$$

where $\boldsymbol{x}_{B_k} > \mathbf{0}_k$.

Brown-Resnick: $W = \exp\left(\tilde{W} - \frac{\sigma^2}{2}\right)$ with $\tilde{W}$ a centered Gaussian process

Extending current classes of max-stable and r-Pareto models

Theorem 2 - skewed Brown-Resnick. [Zhong, Sisson & Béranger (2025)]

Let $W(\mathbf{s}) = \exp\{Y(\mathbf{s}) - a(\mathbf{s})\}$ where $Y(\mathbf{s})$ is a centred skew-normal process with scale matrix Σ with slant parameter α, and $a(\mathbf{s}) = \log \mathbb{E}[\exp\{Y(\mathbf{s})\}]$.

$\Longrightarrow$ The models has <u>no mass on $\partial\Omega$</u>.

Comments:

- The sBR model has a non-stationary dependence structure.
- Removal of the mass on $\partial\Omega$ increases the dependence strength
- Also introduced the truncated extremal-t model.

Extending current classes of max-stable and r-Pareto models

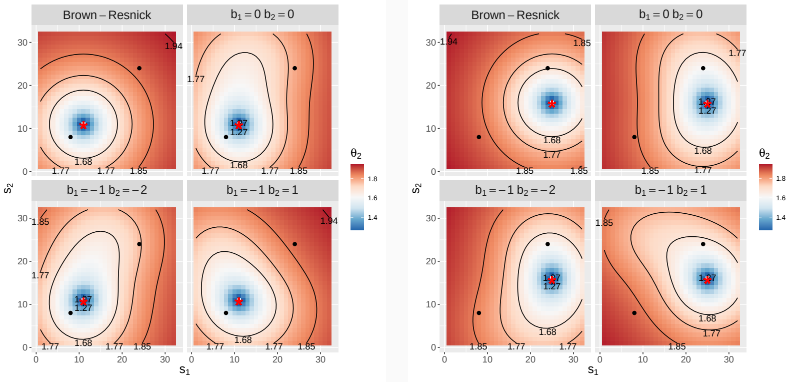

Figure 1: Bivariate extremal coefficient for the Brown-Resnick model and skewed Brown-Resnick model where $\eta_i = \sum_{j=1}^{2} b_j K_j(\boldsymbol{s}_i) + 0.1\mathrm{sgn}(s_{2,i} \geq 16), i = 1, \ldots, D$ using $(b_1, b_2) = (0, 0), (-1, -2)$ and $(-1, -1)$. Black dots denote the kernel centres $\boldsymbol{s}_1^\star, \boldsymbol{s}_2^\star$. A red star indicates the reference point.

Improved inference for r-Pareto models

Where does the idea come from? [Dombry, Legrand & Opitz (2024)]
Using rejection sampling, one can generate samples from a r-Pareto process with risk functional r_2 from samples of a r-Pareto process associated with risk functional r_1 as long as $Mr_1(\cdot) \geq r_2(\cdot), M > 0$.

Focus: Observations $i \in \{m : r(\mathbf{x}_m) > u\}$

Proposal: use the likelihood of the L_1-Pareto process to make inference about any r-Pareto process with a different risk functional by choosing a high threshold $u > M$.

Benefit: Avoids to compute the normalising constant!!

Spectral likelihoods vs score matching

Note. Theorem 3 of Zhong, Sisson & Béranger (2025) provides an efficient algorithm for simulating r-Pareto processes.

Setup:

- Generate $n = 2,000$ obs from the skewed Brown-Resnick model on a 15×15 grid ($D = 225$).
- Power-law semivariogram $\gamma(h) = (h/\lambda)^{\vartheta}$ with range $\lambda = 5, 10$ and smoothness $\vartheta = 1, 1.5$.
- Skewness represented through spline functions with 2 Gaussian kernel basis functions $(b_1, b_2) = (0, 0), (-1, -2), (-1, 1)$.
- L_3 risk functional with exceedances above the 95% empirical quantile.
- 300 replicates.

Spectral likelihoods vs score matching

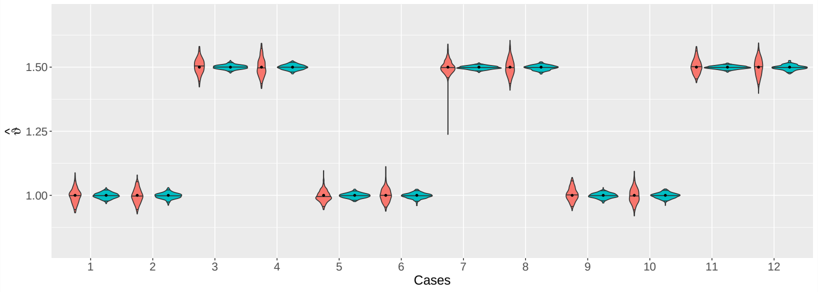

Figure 2: Violin plots for score matching (red) and spectral likelihood (blue) estimates of ϑ for the skewed Brown-Resnick r-Pareto process with L_3 norm risk functional. Black dots indicate the parameter true values.

- The spectral likelihood provides unbiased, low variability estimates.
- The score matching produces unbiased but more variable estimates.

Spectral likelihoods vs score matching

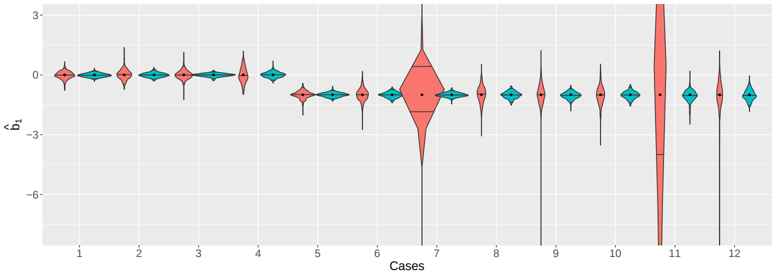

Figure 3: Violin plots for score matching (red) and spectral likelihood (blue) estimates of b_1 for the skewed Brown-Resnick r-Pareto process with L_3 norm risk functional. Black dots indicate the parameter true values.

- Score matching estimates can become numerically unstable (cases 7–12).
- Spectral likelihood is ~ 5 times faster than the score matching approach (141 versus 704 seconds on average using 3 CPU cores).

Analysis of extreme rainfall over Florida

Data:

- Location: Tampa Bay area, Florida. Regular 2km grid with $4,449$ spatial observations.
- Measurements: radar images recorded at 15 minute intervals between 1995–2019 during the wet season (June–September). Total $n = 139,881$ images.
- Smaller version of the dataset analysed in de Fondeville & Davison (2018).
- Risk functions:

  $\rightarrow$ L_∞ norm: defines extremes events as locally intense rainfall events at any location within the region

  $\rightarrow$ L_1 norm selects events with high cumulative rainfall over the whole region.

Analysis of extreme rainfall over Florida

Modelling:

- Brown-Resnick (BR) and skewed Brown-Resnick (sBR) with anisotropic semivariogram.
- Fitting using score matching and spectral likelihood.

Outcomes:

- Brown-Resnick:

  → Spectral likelihood and score matching provide consistent estimates.

  → Spectral likelihood is 80% (L_1 norm) and 18% (L_∞ norm) faster.

- Brown-Resnick vs skewed Brown-Resnick:

  → AIC favours the skewed Brown-Resnick for both L_1 and L_∞ norms.

Analysis of extreme rainfall over Florida

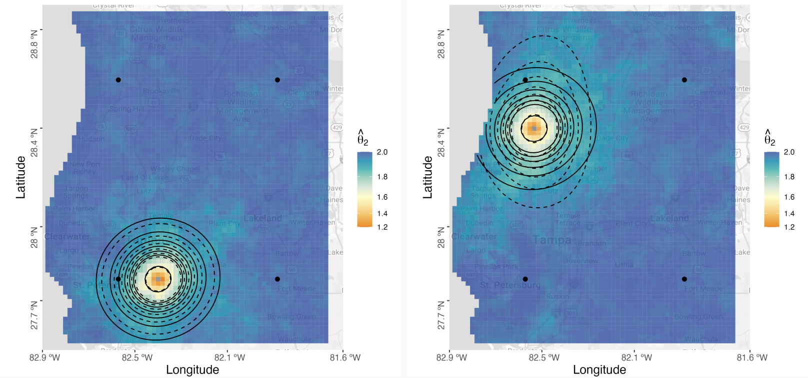

Figure 4: Maps of bivariate empirical extremal coefficients (shading) with respect to two different reference points, and contours of the extremal coefficient of the fitted sBR (dashed line) and BR (solid line) r-Pareto models with L_∞ norm risk functional. Black dots denote the kernel centres used in the sBR model.

Discussion - [Zhong, Sisson & Béranger (2025)]

Contributions:

1. Established condition ensuring the intensity function of a max-stable process only places mass on Ω°, simplifying the evaluation of the density of the r-Pareto process.

2. Likelihood-based inference can be successfully implemented via the spectral likelihood.

3. Two new models: skewed Brown-Resnick and truncated Extremal-t.

4. Improved rejection sampling algorithm for r-Pareto processes.

https://arxiv.org/pdf/2407.13958

Overall discussion

Comments:

- Krupskii & Béranger (2025) defines a factor copula model for extremes.
- Asymmetry in the dependence brings spatial non-stationarity.
- Inference is almost always difficult. Trade-off between model complexity and implementation difficulty.

Some big (and open) questions:

- Are these types of models used by practitioners? Not really, most methods are too difficult to implement.
- Are asymptotic models a good approach to model multivariate/spatial extremes? Yes, if observed in the data...
- Are these r-Pareto models great? Yes and No. Link with max stable processes restrains flexibility.
- Do we have models for (spatial) compound extremes? Needs attention!

THANK YOU